

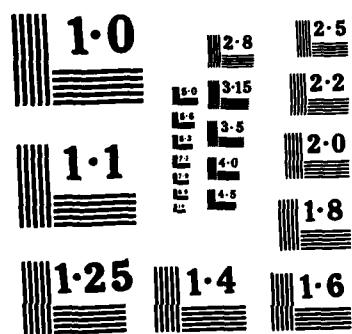
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Martin Shubik

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ENOUGH GOLD IN A SOCIETY  
WITHOUT AND WITH MONEYLENDERS\*

by

Martin Shubik

### 1. QUANTITY AND DISTRIBUTION OF MONEY

If an exchange economy is modeled as a strategic market game with one commodity serving as a money, then if there is no credit available and if all traders are insignificant in size, so that an individual does not influence prices, the noncooperative equilibria (NEs) of the game will coincide with the competitive equilibria of the exchange economy provided that there is enough money to facilitate trade.

The meaning of 'enough money' is that the NEs are interior. In other words the constraint that an individual cannot spend more of the means of payment than he holds is not binding on any individual's plans.

The condition on enough money is characterized both by the total amount of money in the system and its distribution. It is possible that an economy may not have enough money no matter how it is distributed; it

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is also possible that a redistribution will give rise to interior solutions. These statements can be made precise and illustrated by means of specific examples. If there is enough money but it is maldistributed it can be shown that a loan market '100% backed by gold' will bring efficiency.

## 2. EXCHANGE WITHOUT A LOAN MARKET

Let there be  $m$  types of trader trading in  $m+1$  commodities. All traders have the same utility function of the form

$$(1) \quad \theta = \prod_{j=1}^m x_j^{1/m} + x_{m+1} .$$

The separability of the  $m+1$ -st commodity is for ease of calculation in the example and is not critical to the general argument.

Let each trader of type  $i$  have an endowment density of  $(0, 0, \dots, m_k, 0, 0, \dots, 1)$  where the  $m_k$  is for the  $i$ -th good. By inspection the unique CE of this economy is at the symmetric distribution  $(k, k, \dots, k, 1)$  for all. If we set  $p_{m+1} = 1$  then from:

$$(2) \quad \frac{\partial \theta}{\partial x_j} / p_j = 1 \text{ we obtain } \frac{1}{m} \frac{k^{m-1/m}}{k^{m-1/m}} = p_j \text{ or } p_j = \frac{1}{m} .$$

The value of the purchases of each individual equals the value of sales and is  $\frac{1}{m}(m-1)k$ .

If the exchange economy is modeled as a bid offer game (see Dubey and Shubik, 1978) using the  $m+1$ -st commodity as a means of payment then a strategy by a player of type  $i$  is of the form  $(q_1^i, b_1^i, q_2^i, b_2^i, \dots, q_m^i, b_m^i)$  where  $q_j^i \leq a_j^i$  for  $j = 1, \dots, m$  and  $b_j^i \geq 0$  for

$j = 1, \dots, m$  and  $\sum_{j=1}^m b_j^i \leq a_{m+1}^i$ . We omit a subscript or superscript to identify a specific trader. The symbol  $a_j^i$  indicates the amount of good  $j$  held by an individual of type  $i$ . At equilibrium in this game a strategy of a trader of type  $i$  if:

$$(3) \quad \frac{1}{m}(m-1)k \leq 1$$

will be of the form  $(0, k/m, 0, k/m, 0, k/m\dots 0, k/m, (m-1)k, 0, k/m\dots)$  where  $(m-1)k, 0$  is the  $i$ -th pair.

If  $(m-1)k > m$  then no one has enough money for efficient trade.

There will be an NE with constraints which will not coincide with the CE.

We are now in a position to describe a class of different but related games where the only difference in the games is in the distribution of the means of payment. Instead of giving all  $m$  types of trader 1 unit of money each, we give type  $i$ ,  $x^i$  where  $\sum_{j=1}^m x_{m+1}^j = m$ . If  $(m-1)k > m$  then no matter how the money is distributed there is not enough. For every game in this class the NEs will be on the boundary for some traders. This is not true for some of the games if  $(m-1)k \leq m$ .

Whenever there is enough money so that for some distribution there is an interior NE it is easy to construct a distribution with the same amount of money where some individuals will be at a boundary. For example give all traders of one type all of the money. Although the example given above is special the comments so far are completely general, except that it could be possible to select a commodity as a money which could never be in sufficient supply as its marginal worth could fall fast enough to more than offset any increase in supply.

### 3. EXCHANGE WITH A MONEY MARKET

Suppose that there is enough money but the money is badly distributed, for example suppose that traders of one type had all of the money and all the others had no money. Then if we introduce a money market efficiency is restored. We first provide an example, discuss the modeling problems involved in constructing a money market for a playable game and then state the general proposition.

A slightly different model than that used in Section 2 employed here. Let all traders have utility functions of the form

$$(4) \quad \theta = \left( \prod_{j=1}^m x_j^{1/2m} \right) x_{m+1}^{1/2}.$$

If there are  $m$  types of traders where the  $i$ -th type is characterized by an endowment of  $(0, \dots, 0, m_k, 0, \dots, m_k)$  then there is enough money to finance trade. Set  $p_{m+1} = 1$  then at the CE all traders have a final endowment of  $(k, k, \dots, k, m_k)$ ; the prices of all goods will be  $p_j = 1$ . The CE and NE coincide as they all have enough money to finance all trade.\* If instead of distributing the money as above we give it all to traders of type  $m$ , then the distributions are  $(0, 0, \dots, 0, m_k, 0, \dots, 0)$  for type  $i$ ,  $i = 1, \dots, m-1$  and  $(0, 0, \dots, 0, m_k, m^2 k)$  for type  $m$ . In the game where payment must be made in money no trader of type  $i = 1, \dots, m-1$  can trade without credit. In the CE version all traders of type  $i = 1, \dots, m-1$  end up

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\*Note if we had chosen a utility function of the form  $\theta = \prod_{j=1}^{m+1} x_j^{1/m+1}$  then for endowments  $(0, \dots, 0, m_k, 0, \dots, 0, x)$  for traders of type  $i$  no matter what size  $x$  is selected there never would be enough money. The price elasticity for extra money is such that an increase in supply lowers price in a way that purchasing power is never sufficient for the efficient exchange of the other resources.

with  $(ak, ak, \dots, ak; amk)$  and traders of type  $m$  end up with  $((1-a)mk, (1-a)mk, \dots, (1-a)m^2k)$  where  $a = 1/2$ .

If we now introduce a money market where short term financing can be obtained we show that the CE and NE give the same final distributions.

We introduce a two stage game where those with excess money can offer to lend it in the first stage. In the second stage all trade using only money. After trade those who have borrowed redeem their I.O.U. notes for money.

In order to be able to completely specify how borrowing and lending takes place we must describe a mechanism. Four elementary mechanisms are suggested, each of which has an institutional counterpart. The first is pairwise borrowing and lending. A pair of individuals make a private agreement, one gives the other some amount of the monetary commodity in return for a nonnegotiable I.O.U. note which is held by the lender until the borrower  $j$  pays back the loan. Any default arrangement is between  $i$  and  $j$  although society as a whole through the courts may be required for enforcement.

The second is matched pairwise borrowing and lending via a broker. The function of the broker as is indicated in Figure 1b is to serve as an information focus and clearing house so that otherwise anonymous agents can be matched. The broker is not a principal and bears no role after  $i$  and  $j$  have been matched. Figure 1a shows straight matching of lenders and borrowers without any intermediary.

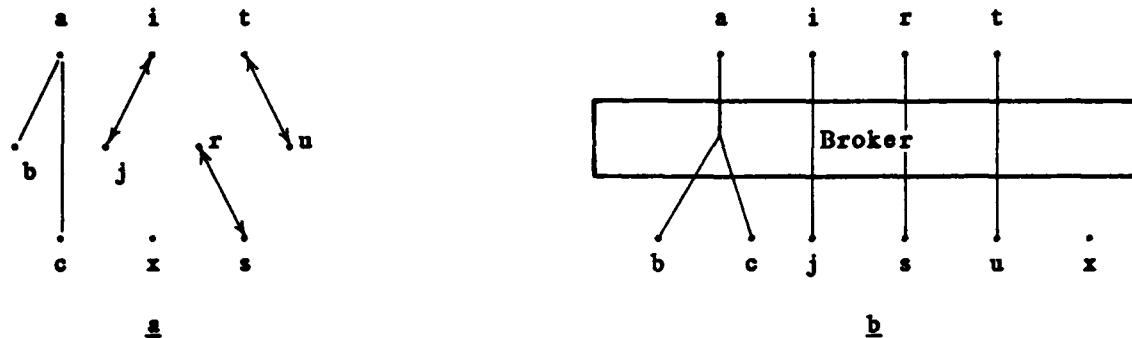


FIGURE 1

The role of the broker is in pooling information and matching principals, not pooling principals. We might expect brokers to fix or to compete for fees. This type of transactions cost is discussed elsewhere (Nti and Shubik, 1984). We note but do not discuss the change from a collection of independent agents to quasi-independent agents in the same club which calls for formal membership and dues, but provides insurance, rule-making, clearing, enforcement of contract, the holding of earnest money and other activities which turn a loosely associated group of individuals into a formal financial institution.

The third possibility (which splits into two subcases) can be described as a rudimentary form of money market or as a commercial bank. Both involve aggregation but there is a difference in who are the principals who bear the responsibility for absorbing losses in the event of a failure to pay back a loan. Figures 2a and 2b help illustrate the distinction. In the rudimentary pooled money market fund the fund serves merely as an aggregating device for both sides of the market. All loanable funds are pooled and then these are matched against the demand for funds to determine a rate of interest. There are two financial instruments involved. Money or 'gold' from the lenders and I.O.U. notes from the borrowers. But these are no longer of the form 'i owes j'

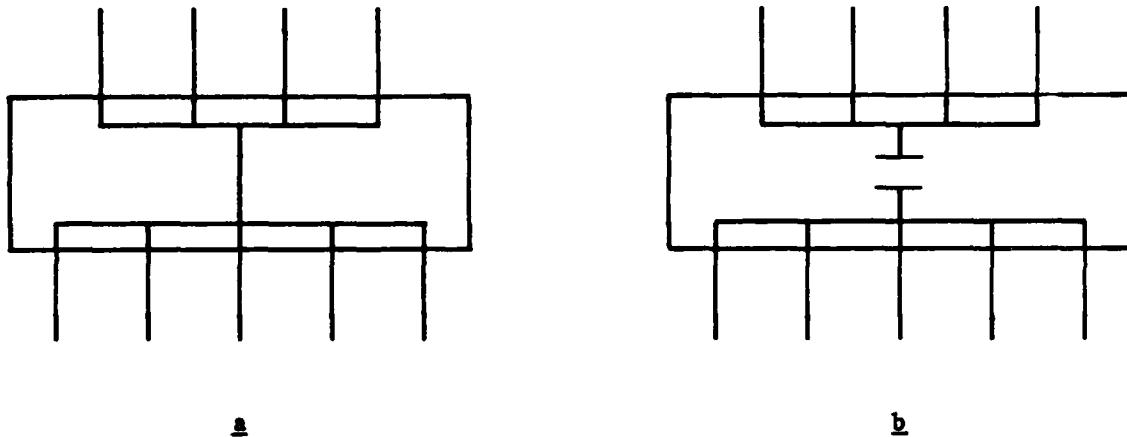


FIGURE 2

but are "' i owes 'the fund.'" If i fails to pay back the fund then the rules of the game require that some form of prorating of the loss be assigned to all creditors. This could involve the specification of seniority, or as in the model here the losses could be required as being in proportion to the amounts involved.

If, instead of modeling a pooled money market where the market is merely an aggregating device which determines the price of money, we wished to model an inside or commercial bank we would need three not two financial instruments. The first is the commodity money or gold deposited by the agents with surplus gold (they can be called, lenders, savers or depositors). The second is the I.O.U. note given by the bank to these individuals. 'The bank owes individual i .' The third instrument is the I.O.U. note between borrowers and the bank 'Individual j owes the bank.' The problem of 100% reserve banking appears at this point in the rules. Does the bank issue paper or gold to the borrowers? If it issues gold or is required to issue 100% gold backed paper then (leaving aside any capital of the bankers: see the Appendix for a discussion of bank reserves) it can only issue as loans up to its deposits. If this is the

case then the difference between this model and the money market model is that one more financial instrument was created and that liability has been changed. We need not only bankruptcy rules but ban failure rules because if a borrower fails to pay the bank this may cause the bank to fail against the depositors. In a world without exogenous uncertainty this may appear to be much ado about nothing as in a well designed system there is no reason for anyone to default, but the complete definition is required for a playable game.

A fourth model can be constructed by imagining a central or government bank which issues 'paper gold' in return for I.O.U. notes of the individuals. Unless the government itself had gold this cannot be a 100% reserve system but involves the creation of government money. Shubik and Wilson (1977) and Dubey and Shubik (1979) have studied this model in some detail.

Here we confine ourselves to the third model which is that of a money market fund. Before the general proposition is noted we continue the example modifying the utility function given in (4) to account for the valuation of the holdings (positive or negative) of I.O.U. notes.

Beyond the  $m+1$  commodities, we introduce one new financial instrument, the I.O.U. note. A trader of type  $i$  who wishes to borrow, bids an amount  $b_{m+2}^i$  which is interpreted as follows. The specific trader promises to pay the market at the end of all trade an amount  $b_{m+2}^i$  in gold. The trader of type  $j$  who wishes to lend offers the market  $q_{m+1}^j$  in gold. The price of a loan is fixed by:

$$(5) \quad \frac{\sum_{i=1}^m b_{m+2}^i}{\sum_{j=1}^n q_{m+1}^j} = 1 + \rho .$$

This states that one plus the rate of interest is determined by dividing the sum of all promises to pay by the sum of all loanable funds offers.

In the game which has a pooled money market and a continuum of traders a strategy becomes:

$$(6) \quad (b_1^i, q_1^i, \dots, b_m^i, q_m^i, q_{m+1}^i, b_{m+2}^i)$$

where the first  $m$  pairs represent a bid of an amount of money and an offer of an amount of good  $j$  in the market  $(j, m+1)$  where  $j$  is exchanged for money. The final market  $(m+1, m+2)$  is where gold is exchanged for I.O.U. notes.

As I.O.U. notes can be created at will by those who wish to borrow money we face a game design problem concerning the unbounded creation of debt. In actuality if a private firm or individual goes to a bank or any other source of borrowing, his I.O.U. note becomes more and more suspect as its size grows in relation to the real resources he owns.

A complicated game could distinguish between 'prime names' and 'lesser names' in their ability to create debt, but this is not a logical necessity.

In order to keep the game well defined and playable we need to introduce rules which have the property that they have the effect of bounding rational strategic behavior. This means that if an individual acting in a rational optimizing manner were informed of the actions of all others, no matter how irrational or bizarre, his best reply would always be bounded in the amount of I.O.U. notes he issues.

A natural way to keep the amount of debt issued bounded is to introduce penalties for failure to fully redeem one's I.O.U. notes at the

end of the game. Clearly if the promises to pay in gold amount to more gold than there is in the whole economy some individuals will be unable to meet their full obligations.

When a default occurs society must make adjustments concerning two sets of individuals, the debtors and the creditors. Thus discouragement of default or punishment of defaulters is only part of the problem. The rules must be such that lenders are not discouraged from lending and furthermore if a default occurs there must be a specification of the type of arrangements to be made. These could be 'Ten cents in the dollar,' the right to confiscate the debtor's remaining property or many other noneconomic but societally sanctioned arrangements.

There appear to be two fundamentally different situations involving default. The first is where there is no exogenous uncertainty present in the economy as is the situation here. The second is where there is exogenous uncertainty present. In the first instance society need only guard against purposeful strategic default. It is a problem of moral hazard. The individual goes into default because it pays to do so.

When there is exogenous uncertainty present and the money market treats all individuals in aggregate regardless of the possibility that they may have different risk aversion the possibility of default involves luck as well as intent. A Draconian default penalty could penalize borrowers so badly that no one would dare to borrow. Too light a penalty could cause the lenders to refuse to lend. If penalties are not designed taking into account individual risk attitudes then the selection of a single anonymous penalty is the equivalent to the creation of a public good which sets the expected acceptable level of default.

When no exogenous uncertainty is present, if a single anonymous

penalty is sufficiently harsh to discourage strategic default for all; then creditors do not need to be concerned with seniority of claims and settlement arrangements. There will be no default in equilibrium.

Even though a rational solution will not involve default when there is no exogenous uncertainty, a playable game (as contrasted with its solution) requires the specification of settlement rules in case of default.

In two previous papers Shubik and Wilson (1977) and Dubey and Shubik (1979) have discussed the optimal default punishment for any economy without exogenous uncertainty where individuals can borrow fiat money from an outside or government bank. In essence the problem is the same here as far as a borrower is concerned.

An easy way to construct an optimal default penalty for the economy with a money market is to imagine that at the end of trade, when the I.O.U. notes are due, all individuals pay up whatever they can and the debt still outstanding is transmuted into holdings of negative amounts of gold. Thus we adjust the definition of the utility function so that it is defined not merely in  $R_{m+1}^+$  but in  $R_m^+ \times R$ . We consider the utility of negative final holdings of money directly. Figure 3 shows the indifference curves for gold drawn against an aggregate bundle of all other goods. Negative gold is I.O.U. notes in default.

The continuation of the convex contours of the indifference curves into the negative orthant indicates in a general way how badly going into default hurts. The line  $M_1 I_1 E_1$ , is to be interpreted as follows. Suppose that the price system at an equilibrium is known. The value of an individual's initial resources are indicated by  $I_1$ . In terms of gold they are worth  $M_1$ . An individual facing this price system is willing to choose a strategic default at point  $E_1$  where he defaults a general

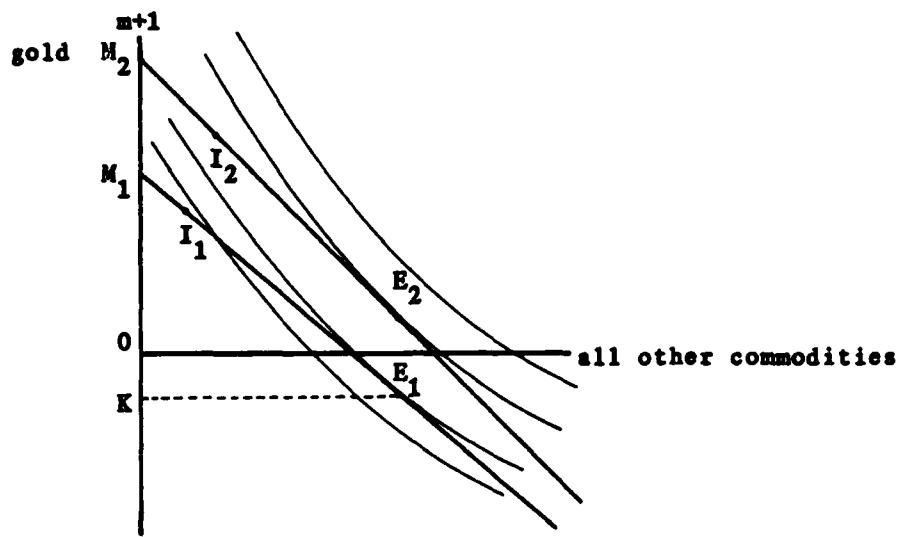


FIGURE 3

equilibrium system with initial resources of  $I_2$  and prices indicated by  $M_2, I_2, E_2$  would not default but would trade to  $E_2$ .

If computation were costless the referee (government) in this game could pick precisely the least harsh default penalty to guarantee that any CE in the exchange economy could be attained as an NE of the strategic market game. Solve the exchange economy for its set of CEs, select any CE you wish to have as an NE of the game. There are  $m$  types of traders hence at the CE there will be  $m$  different Lagrangian multiplier  $(\lambda_1, \lambda_2, \dots, \lambda_m)$  where each can be interpreted as the marginal utility of an extra unit of gold at equilibrium. Select the maximum of these:

$$(7) \quad \lambda^* = \max(\lambda_1, \lambda_2, \dots, \lambda_m).$$

We now extend the definition of the utility functions of all individuals as follows:

$$(8) \quad \begin{aligned} \theta_i &= \phi_i(x_1^i, \dots, x_{m+1}^i) \text{ for all } x_j^i \geq 0, \quad j = 1, \dots, m+1 \\ &= \phi_i(x_1^i, \dots, x_m^i, 0) + \lambda^* x_{m+1}^i \text{ when } x_{m+1}^i \leq 0. \end{aligned}$$

This merely states that the marginal disutility of default is always greater than or equal to the marginal utility of income at equilibrium.

As neither computation or knowledge are free the referee may not be able to carry out this calculation. Fortunately when no exogenous uncertainty is involved he does not need to calculate. All he needs to do is to make the penalty harsh enough (Schmeidler, 1976, made it infinite). This is not true when exogenous uncertainty is present.

We now complete our example. The full game is as indicated. In extensive form it is shown in Figure 4.

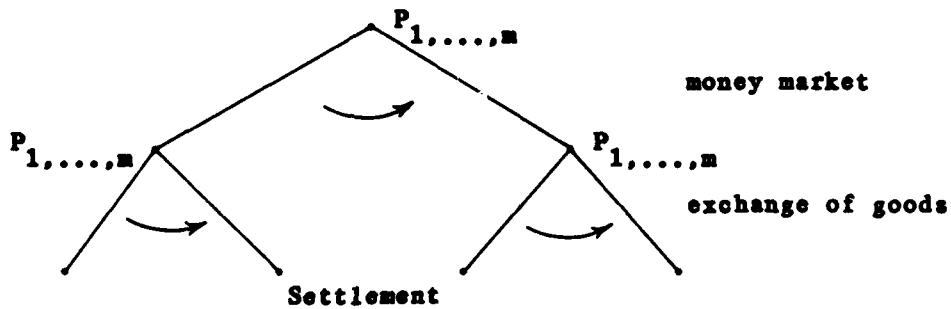


FIGURE 4

The label  $P_{1,\dots,m}$  indicates that all players of all types move simultaneously in the money market. The arrows indicate that the strategies are continuous and cannot be fully described by a finite tree. The information conditions indicate one element sets, or perfect information. In a game without exogenous uncertainty and nonatomic players the distinction between perfect equilibria and non perfect equilibria disappears (see Dubey and Shubik, 1981). In essence if you know something but all others choose to ignore the knowledge, if you are too small to influence the market it does not do you any good. A bargain that remains a bargain is no bargain (Shubik and Whitman, 1979). The

important technical observation is that strategies in this two stage game, even with information do not become complex functions of the previous moves of everyone but depend only upon the state you are in. However even the strategies which are simple vectors as shown in (6) will yield perfect NEs.

Suppose that the utility function to all players is

$$(10) \quad \begin{aligned} \theta &= \left( \prod_{j=1}^m x_j^{1/2m} \right) x_{m+1}^{1/2} \quad \text{for } x_{m+1} > 0 \\ &= -\lambda^* x_{m+1} \quad \text{for } x_{m+1} \leq 0 \quad \text{where } \lambda^* \text{ is any positive constant.} \end{aligned}$$

The initial endowments are as before  $(0, \dots, 0, m_k, 0, \dots, 0)$  for traders of type  $i$ ,  $i = 1, \dots, m-1$  and  $(0, \dots, 0, m_k, m^2 k)$  for the monied traders and traders of type  $m$ .

The settlement procedure in case of default is that whatever money is collected from the borrowers is paid back to the lenders in proportion to their claims. All lenders have the same seniority and share the defaults.

We may check that the following strategies form an NE which coincides with the CE of the exchange economy. A trader of type  $i$  ( $\neq m$ ) selects  $(b_1^i, 0, b_2^i, 0, \dots, 0, q_1^i, \dots, b_m^i, 0, b_{m+2}^i, 0)$  where the amount  $b_{m+2}^i = \left[\frac{m-1}{2}\right]k$  is the I.O.U. note he bids to obtain gold. The  $b_j^i = \frac{m-1}{2m}$  are the percentages of the gold he obtains offered for commodity  $j = 1, \dots, m$  and  $j \neq i$ .  $q_j^i = \left[\frac{2m-1}{2}\right]k$  is the amount of the  $i$ -th good offered for sale by traders of type  $i$ . The strategy of a trader of type  $m$  is given by  $(b_1^m, 0, \dots, b_{m-1}^m, 0, 0, q_m^m, 0, q_{m+1}^m)$  where  $b_j^m = \frac{mk}{2}$ ,  $j = 1, \dots, m-1$  is the amount of money he spends on item  $j = 1, \dots, m-1$ ,  $q_m^m = \frac{mk}{2}$  the amount of  $m$  offered for sale and  $q_{m+1}^m = \frac{m(m-1)}{2}k$  is the amount of money offered to the loan market,

$$(11) \quad \sum_{j=1}^{m-1} b_j^m \leq a_{m+1}^m - q_{m+1}^m .$$

We may check that if  $p_{m+1} = 1$  is set in advance then at the CE all  $p_j = 1$ . Furthermore from the strategies

$$1+\rho = \frac{m((m-1)/2)k}{m(m-1)k/2} = 1 \text{ hence } \rho = 0 .$$

Thus competition among lenders enables the borrowers to obtain the bridging finance they need between when they buy and when they are paid for what they have sold, at a zero rate of interest. The CE is an NE as given prices  $p_j = 1$  and  $\rho = 0$  and the default penalty these strategies are best responses. The rate of interest would not be zero if there were a time discount reflected in the utility functions to distinguish between the possible change in valuation of goods before and after trade.

Another factor would produce a positive rate of interest and a non-Pareto optimal outcome. Suppose that there were only one or two individuals of type  $m$  rather than a continuum. Then it is relatively easy to check that they might hold back on loanable funds. With few moneylenders the anonymous money market though logically possible is not as plausible as an oligopolistic model with moneylenders or lending banks which may lend only their own money. In a previous paper (Shubik, 1976) an example was given of duopolistic banks issuing paper money, but with slight modification these results would apply to two banks or moneylenders with all of the gold.

The discussion so far has been carried out and illustrated by means of examples, but the results can be stated quite generally.

Let  $E(n, \delta, a^i, x_{m+1}^i \geq 0, \sum_{j=1}^n x_{m+1}^j = a_{m+1})$  be the class of exchange economies with  $n$  types of traders. All traders of type  $i$  have the same endowment and utility function  $(a_1^i, \dots, a_m^i, x_{m+1}^i)$  and  $\delta^i$ .

Associated with every member of the class  $E$  is a strategic market game

with a money market  $\Gamma_c(n, \tilde{\delta}, \lambda^*, a^i, x_{m+1}^i \geq 0, \sum_{j=1}^n x_{m+1}^j = a_{m+1})$  where

$\lambda^*$  is the default penalty parameter,\*  $\tilde{\delta}$  is  $\delta$  modified for negative values and the  $m+1$ -st good is used as the means of payment. Let  $\Gamma$  be the one stage class of games with no money market. As there is no credit granted in  $\Gamma$  there is no need to introduce a default penalty.

Associated with every exchange economy of  $E$  there will be a set of CEs. We are now in a position to compare each exchange economy in  $E$  with an associated strategic market game in  $\Gamma$  (without a money market) and in  $\Gamma_c$  (with a money market). Table 1 shows the six cases involving  $\Gamma$  and  $\Gamma_c$ .

Consider a specific exchange economy in  $E$  which has a price system at a CE denoted by  $p_1, \dots, p_m, 1$ . (We set  $p_{m+1} = 1$ ) and a distribution to trader type  $i$  of  $(x_1^i, \dots, x_{m+1}^i)$  at the CE.

The inequalities [A] are individual cash flow constraints. They state that each individual must have enough money to finance his purchases. The amount  $(x_j^i - a_j^i)$ , when positive is a purchase.

The inequality [B] does not depend on the distribution of money, but is 'an enough gold' constraint. Does society as a whole have enough gold in aggregate to finance trade?

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\*Selected to be at least as large as the largest marginal utility for money at every CE in the class of exchange economies, where  $p_{m+1} = 1$  is the numeraire.

$\Gamma$	$\Gamma_c$
<p>1 The inequalities</p> <p>[A] <math>\sum_{j=1}^n p \cdot \max[0, (x_j^i - a_j^i)] \leq a_{m+1}^i</math></p> <p>The CE coincides with an NE.</p>	<p>2 Inequalities [A] hold.</p> <p>The CE coincides with an NE and no credit is needed.</p>
<p>3 The inequality</p> <p>[B] <math>\sum_{i=1}^n \sum_{j=1}^m p \cdot \max[0, (x_j^i - a_j^i)] \leq \sum_{i=1}^n a_{m+1}^i</math> is satisfied, but all of [A] are not satisfied. The CE does not coincide with an NE.</p>	<p>4 The inequality [B] holds.</p> <p>The CE coincides with an NE and 100% gold backed money market is active.</p>
<p>5 The inequality [B] is not satisfied. The CE does not coincide with an NE.</p>	<p>6 The inequality [B] is not satisfied. There is not enough gold for 100% backed credit. The CE does not coincide with the NE. Fiat money and the appropriate default rules are needed for the CE, NE coincidence.</p>

TABLE 1

Theorem: If for an exchange economy in  $E$  and the related strategic market game  $\Gamma_c$ , at a CE the following inequality is satisfied

$$\sum_{i=1}^n \sum_{j=1}^m p \cdot \max[0, (x_j^i - a_j^i)] \leq \sum_{i=1}^n a_{m+1}^i \text{ then the CE of } E \text{ coincides with an NE of } \Gamma_c .$$

Proof: The proof is in essence a reinterpretation of the proof given in Dubey and Shubik (1979) for credit supplied by an outside bank.

#### 4. DISCUSSION

The example calculated in Section 3 has the property that inequality [B] holds for all games related to  $E$ , but the inequality [A] holds for only some of them. In general however as the price system may shift considerably with the reallocation of a commodity money there is no guarantee that [B] will hold for all games.

The concept of enough commodity money involves the total amount available, its distribution and its velocity.\* The question of velocity has been avoided in this model. Holding velocity fixed it has been shown that trade using gold will be efficient with a 100% gold backed loan market if there are no oligopolistic elements and if there is enough gold. Mathematically enough gold means that inequality [B] is satisfied.

#### APPENDIX

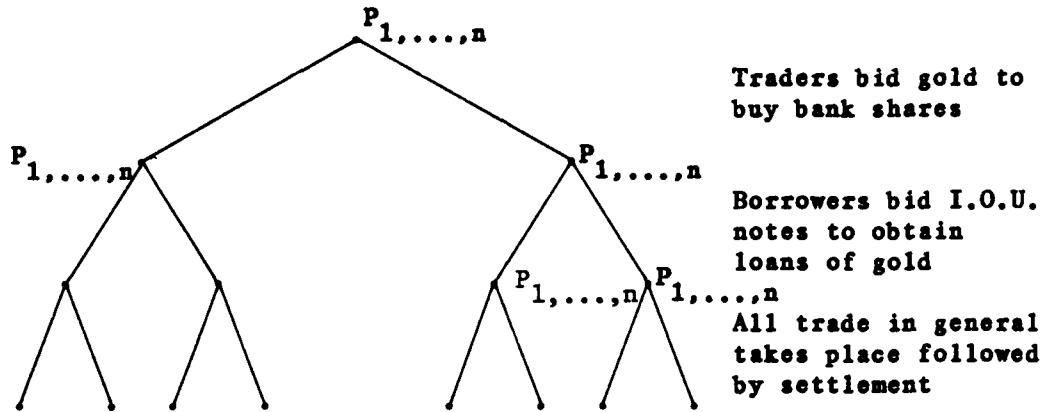
##### On One Hundred Percent Reserve Banking

A discussion of 100 percent reserve banking is given here in an appendix as it requires noting but is an aside to the main discussion. The strict meaning of 100 percent reserve banking is that the loans are bounded by the capital of the owners, not the deposits. In order to model this structure and produce a playable game we would need to introduce a further financial instrument, some form of ownership paper or equity. The common stock is one such instrument. There are several institutionally different structures we could build, but without

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\*There is also the possibility that the number of markets is more than  $m$  but less than  $m(m+1)/2$ , there may be a single money but some nonmonetary trade takes place, as is the case in many developing economies where the nonmonetary sector is considerable. The presence of extra markets lessens the need for money and it is possible to rewrite the inequalities [A] and [B] to take them into account.

pretending to specific institutional realism one is specified which could be played as a simple game, which at the same time highlights several difficulties in bank regulation and competition. Figure 5 shows the extensive form of the game:



**FIGURE 5**

As can be seen from the extensive form the first set of moves is for all traders to bid simultaneously for bank shares. In the second move the bank offers all of its capital on loan. The competitive bidding of I.O.U. notes determines the rate of interest and rations the loans of gold. Then general trade takes place and is followed by settlement.

Several features of this mechanism must be noted. Because only the capital of the bank is loaned it can never fail hence bank failure rules are not required. In the model suggested here there is only one inside or commercial bank. How can we prevent it from exerting monopoly power? Furthermore who runs it? Viewing it as a playable game there is a simple (but by no means unique) answer. By the rules of the game the bank is passive it is a dummy player with a single preset strategy. It passively

auctions its common stock, then it auctions in the loan market whatever it has obtained as capital. By imposing these rules on the bank problems concerning monopoly power and fiduciary responsibility of the managers are avoided. A discussion of the problem of competitive banking is deferred to a projected paper.

With the model above it is straightforward to check that a zero rate of interest with efficient lending emerges. The commercial needs of trade are financed. If a multistage model with a time discount were introduced then the interest rate in equilibrium would not be zero.

## REFERENCES

Dubey, P. and M. Shubik, The Noncooperative Equilibria of a Closed Trading Economy with Market Supply and Bidding Strategies, Journal of Economic Theory 17, 1 (February 1978), pp. 1-20.

\_\_\_\_\_, Bankruptcy and Optimality in a Closed Trading Economy Modelled as a Noncooperative Game, Journal of Mathematical Economics, 6 (1979), pp. 115-134.

\_\_\_\_\_, Information Conditions, Communication and General Equilibrium, Mathematics of Operations Research, 6, 2 (May 1981), pp. 186-189.

Nti, K. O. and M. Shubik, Noncooperative Exchange Using Money and Broker-Dealers, International Journal of Mathematical Social Sciences, 7 (1984), pp. 59-82.

Schmeidler, D., unpublished paper, presented 1976.

Shubik, M., A Noncooperative Model of a Closed Trading Economy with Many Traders and Two Bankers, Zeitschrift fur Nationalökonomie 36 (1976), pp. 49-60.

\_\_\_\_\_, and C. Wilson, The Optimal Bankruptcy Rule in a Trading Economy Using Fiat Money, Zeitschrift fur Nationalökonomie, 7 (1978), pp. 337-354.

Shubik, M. and M. Whitman, The Aggressive Conservative Investor. New York: Random House, 1979.

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